

# WAVELETS AND PRONY METHOD FOR DISTURBANCE DETECTION IN WIND FARMS

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**Abstract** – Wind generator impact on power quality is crucial for power system operation. This paper presents assessment of transients originated in a power system with fixed speed wind farm. Faults in the power system and switching capacitor bank cause transients. Detailed power quality studies on transients are helpful in better assessment of the phenomena. Prony method and wavelets have been applied to determine signal parameters in different operation modes of wind generation unit.

**Keywords:** wind power generation, transients' analysis, wavelets, Prony method.

## 1 INTRODUCTION

Wind generation is one of the most important renewable energy technologies [1]. The connection of wind generators leads to many disturbances, such as: voltage fluctuation, flickers, harmonics, loss of stability, blind power regulation problems, and transients [2, 5].

Power quality issues connected with wind generation are not only important because of technical aspects, they have also influence on the free energy market. Traditionally, wind turbines were disconnected from the grid when an abnormal grid voltage occurred at wind terminals. The reason was due to the fact that under fault conditions there is an increase of the current in the stator windings of the induction generator. Grid faults generate typically transients in the generator electromagnetic torque, which result in significant stress of the wind turbine

If wind farms are unable to withstand voltage drops for a limited time, they will be disconnected from the system and that may cause a cascading voltage fall and the breakdown of part or entire power system.

Compensating capacitors are connected at fixed-speed wind turbines to compensate reactive power consumed by the generator. During the switching of capacitors transients occur, which are devastating for sensitive equipment, protection relays and insulation. The impact of transients on power quality indices cannot be neglected [2, 6, 8].

The precise evaluation of these problems is crucial for appropriate proposal of mitigation systems. One possibility for mitigating voltage dips is to use a Custom Power system such as Dynamic Voltage Restorer DVR connected at wind farm installations. For mitigating transients one possibility is to first magnetize the gen-

erator by capacitors and then synchronize it to the grid. Another possibility is to use power electronic switches to connect the capacitors.

Nonetheless, the first step is the precise evaluation of signal parameters (voltage dip or transient) and afterwards the assessment of problem severity. The authors propose an alternative method to DFT for disturbance detection.

The Prony method [3, 4, 10] was considered as an appropriate tool for parameters estimation of transients. Fourier method was also applied and compared with Prony method. Precise computation of the starting point of a disturbance was done using wavelets [7].

The preliminary research results show advantages of the method based on the Prony model over the Fourier technique.

## 2 PRONY METHOD

The Prony method [10] is a technique for modelling sampled data as a linear combination of exponential functions. Although, it is not a spectral estimation technique, the Prony method has a close relationship to the least squares linear prediction algorithms used for AR and ARMA parameter estimation. Prony method seeks to fit a deterministic exponential model to the data in contrast to AR and ARMA methods that seek to fit a random model to the second-order data statistics.

Assuming  $N$  complex data samples, the investigated function can be approximated by  $p$  exponential functions:

$$y[n] = \sum_{k=1}^p A_k e^{(\alpha_k + j\omega_k)(n-1)T_p + j\psi_k} \quad (1)$$

Where:

$n = 1, 2, \dots, N$ ,  $T_p$  - sampling period,  $A_k$  - amplitude,  $\alpha_k$  - damping factor,  $\omega_k$  - angular velocity,  $\psi_k$  - initial phase.

The discrete-time function may be concisely expressed in the form:

$$y[n] = \sum_{k=1}^p h_k z_k^{n-1} \quad (2)$$

Where:

$$h_k = A_k e^{j\psi_k}, \quad z_k = e^{(\alpha_k + j\omega_k)T_p}$$

The estimation problem is based on the minimization

of the squared error over the  $N$  data values.

$$\delta = \sum_{n=1}^N |\varepsilon[n]|^2 \quad (3)$$

Where:

$$\varepsilon[n] = x[n] - y[n] = x[n] - \sum_{k=1}^p h_k z_k^{n-1} \quad (4)$$

This turns out to be a difficult nonlinear problem. It can be solved using the Prony method that utilizes linear equation solutions.

If as many data samples are used as there are exponential parameters, then an exact exponential fit to the data can be made.

Consider the  $p$ -exponent discrete-time function:

$$x[n] = \sum_{k=1}^p h_k z_k^{n-1} \quad (5)$$

The  $p$  equations of (5) may be expressed in matrix form as:

$$\begin{bmatrix} z_1^0 & z_2^0 & \dots & z_p^0 \\ z_1^1 & z_2^1 & \dots & z_p^1 \\ \vdots & \vdots & & \vdots \\ z_1^{p-1} & z_2^{p-1} & \dots & z_p^{p-1} \end{bmatrix} \cdot \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_p \end{bmatrix} = \begin{bmatrix} x[1] \\ x[2] \\ \vdots \\ x[p] \end{bmatrix} \quad (6)$$

The matrix equation represents a set of linear equations that can be solved for the unknown vector of amplitudes.

Prony propose to define the polynomial that has the exponents as its roots:

$$F(z) = \prod_{k=1}^p (z - z_k) = (z - z_1)(z - z_2) \dots (z - z_p) \quad (7)$$

The polynomial may be represented as the sum:

$$\begin{aligned} F(z) &= \sum_{m=0}^p a[m] z^{p-m} \\ &= a[0] z^p + a[1] z^{p-1} + \dots + a[p-1] z + a[p] \end{aligned} \quad (8)$$

Shifting the index on (5) from  $n$  to  $n-m$  and multiplying by the parameter  $a[m]$  yields:

$$a[m] x[n-m] = a[m] \sum_{k=1}^p h_k z_k^{n-m-1} \quad (9)$$

Equation (9) can be modified into:

$$\begin{aligned} \sum_{m=0}^p a[m] x[n-m] &= \\ &= \sum_{k=1}^p h_k z_k^{n-p} \left\{ \sum_{m=0}^p a[m] z_k^{p-m-1} \right\} \end{aligned} \quad (10)$$

The right-hand summation in (10) may be recognized as a polynomial defined by (8), evaluated at each of its roots yielding the zero result:

$$\sum_{m=0}^p a[m] x[n-m] = 0 \quad (11)$$

The equation can be solved for the polynomial coefficients. In the second step the roots of the polynomial defined by (8) can be calculated. The damping factors

and sinusoidal frequencies may be determined from the roots  $z_k$ .

For practical situations, the number of data points  $N$  usually exceeds the minimum number needed to fit a model of exponentials, i.e.  $N > 2p$ . In the overdetermined data case, the linear equation (11) should be modified to:

$$\sum_{m=0}^p a[m] x[n-m] = e[n] \quad (12)$$

The estimation problem is based on the minimization of the total squared error:

$$E = \sum_{n=p+1}^r |e[n]|^2 \quad (13)$$

### 3 WAVELET TRANSFORM

Wavelets can be applied for precise computation of the beginning of a disturbing event.

The continuous *Wavelet Transform* (CWT) of a signal  $x(t)$  is defined as:

$$X_{a,b} = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{+\infty} x(t) \psi\left(\frac{t-b}{a}\right) dt \quad (14)$$

where  $\psi(t)$  is the mother wavelet (15), and the other wavelets are its dilated and translated versions, where  $a$  and  $b$  are the dilation parameter and translation parameter, respectively,  $a \in R^+ - \{0\}$ ,  $b \in R$  [7].

$$\psi_{a,b}(t) = \left(\frac{1}{\sqrt{a}}\right) \psi\left(\frac{t-b}{a}\right) dt \quad (15)$$

In the practice the discrete WT (DWT) is used. Calculations are made for chosen subset of scales and positions. This scheme is conducted by using filters and computing so called *approximations and details*. The *approximations* ( $A$ ) are the high-scale, low frequency components of the signal. The *details* ( $D$ ) are the low-scale, high-frequency components. The DWT coefficients are computed using the equation:

$$X_{a,b} = X_{j,k} = \sum_{n \in Z} x[n] g_{j,k}[n] \quad (16)$$

where  $a = 2^j$ ,  $b = k2^j$ ,  $j \in N$ ,  $k \in Z$ .

The wavelet filter  $g$  plays the role of the mother wavelet. The decomposition (filtering) process can be iterated, so that one signal is broken down into many lower resolution components. This is called the *wavelet decomposition tree*. For detection of transients a multi-resolution analysis tree (Fig. 1), based on wavelets has been applied.

Every one of wavelet transform subbands is reconstructed separately from each other, so as to get  $k+1$  separated components of a signal  $x[n]$ . The MATLAB *multires* function [9] calculates the approximation to the  $2^k$  scale and the detail signals from the  $2^1$  to the  $2^k$  scale for a given input signal. It uses the analysis filters  $H$  (lowpass) and  $G$  (highpass) and the synthesis filters  $RH$  and  $RG$  (lowpass and highpass respectively) (Fig. 2).

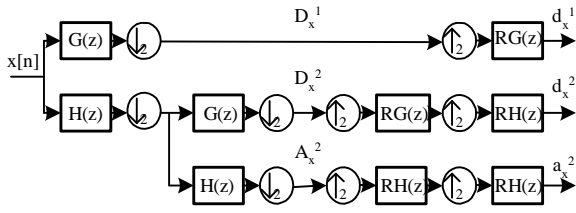


Figure 1: Analysis-Synthesis tree

For signal analysis, the Daubechies D6 wavelet has been chosen as example. There was no significant difference for D4 or D8. Higher order however e.g. 12 may cause longer responses times. Daubechies wavelets are a family of orthogonal wavelets, defining a discrete wavelet transform and characterized by a maximal number of vanishing moments for some given support. With each wavelet type of this class, there is a scaling function which generates an orthogonal multiresolution analysis.

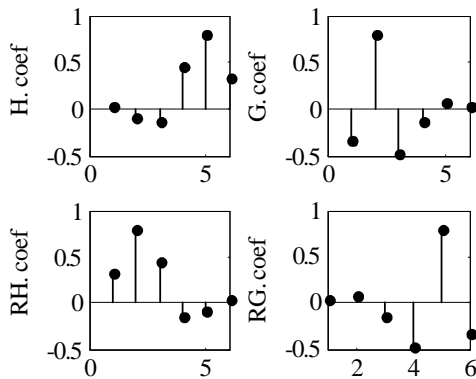


Figure 2: Filters coefficients for Daubechies D6 wavelet.

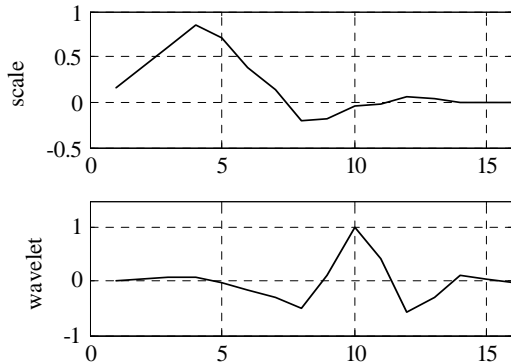


Figure 3: Daubechies D6 scale and wavelet functions.

#### 4 SIMULATION OF WIND POWER STATION

In order to test the performance of the algorithms proposed in this paper for voltage dip and transient detection, some simulations have been done.

The diagram of power system with a wind generator and compensating capacitors is shown in Fig.4.

In the simulations performed, the wind farm is composed by fixed-speed wind turbines where each electrical machine consists of an induction generator of 690V, 750kW and bank capacitors for reactive power compen-

sation. The distribution line is represented by its  $\Pi$  equivalent circuit.

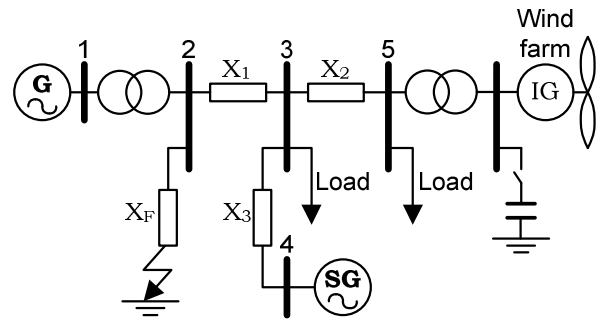


Figure 4: Diagram of a power system with induction generator and compensating capacitors

The wind farm is assumed to be formed by  $n$  generators in parallel, all with similar characteristics. All the simulations were done assuming that the whole wind farm is operating at 0.99 leading PF. For simplicity, it has been assumed that each of the turbines experiences the same wind speed. Therefore, the whole farm may be represented by its equivalent induction generator. The main parameters are given in Table 1.

Wind Farm rating	20MVA
Wind farm voltage	690V
Supply frequency	50 Hz
Wind farm bank capacitors	0.2 p.u. - 0.6 p.u.
Wind farm transformer	20kV/690V, X <sub>cc</sub> =5% 20MVA, YNd11
Transmission lines	(0.01+j0.05) p.u.
Synchronous generator G	20 MVA
Synchronous generator SG	80 MVA

Table 1: System parameters.

A wind turbine generates power accordingly to mechanical torque on the rotating shaft of the turbine. Produced power depends on rotor speed and pitch angle and is often given in a table form. Typical turbine characteristics for different wind speed are shown in Fig. 5.

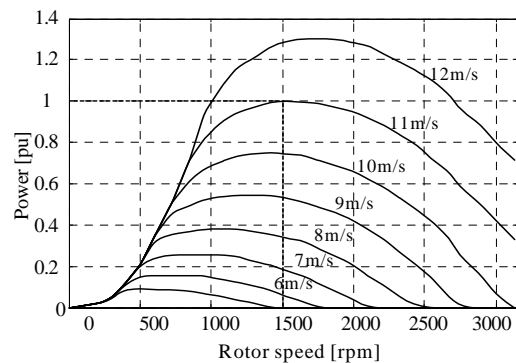
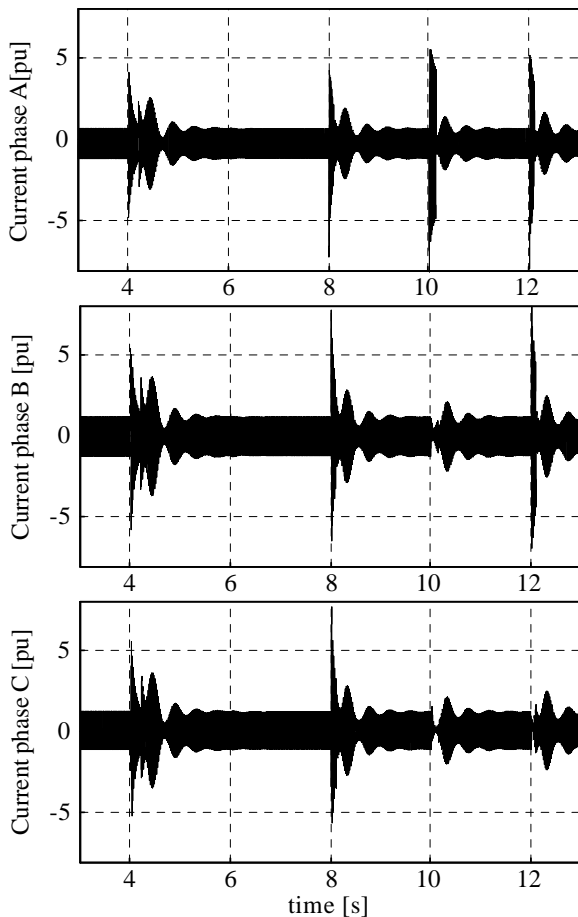


Figure 5: Induction generator output power vs. angular rotor velocity, for different wind speed.

During the research different operation conditions, especially faults have been simulated (Table 2). Simulation results were correlated with measurements. The signal was sampled at 10 kHz.

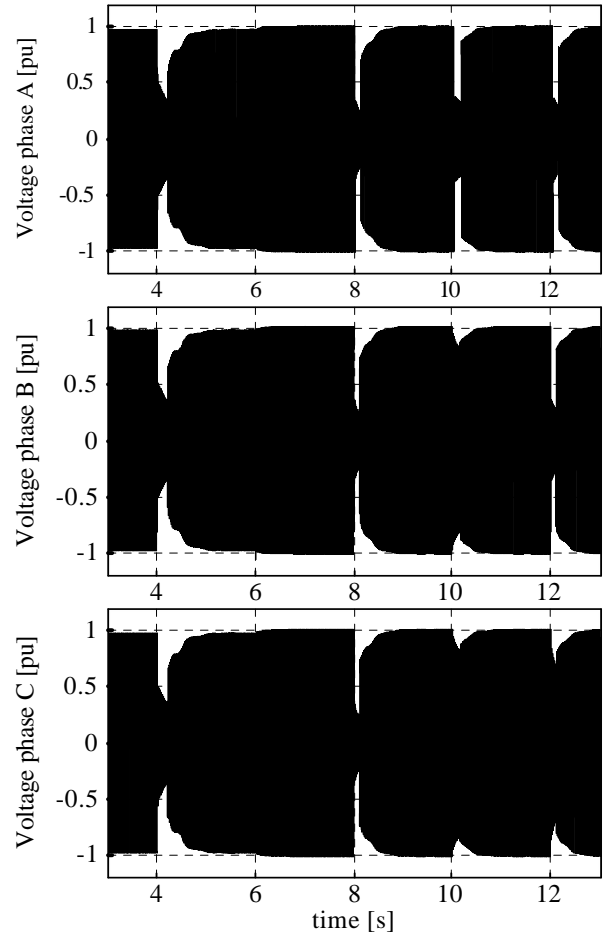
Fault scenarios	$t_{start}$ (s)	$t_{end}$ (s)	$Z_{fault}$ (p.u)
Three phase short circuit bus #2	4	4,2	0
Bank capacitor switching at wind farm	6	-----	-----
Three phase to earth short circuit bus #2	8	8,1	0.05
Single phase to earth fault bus #2	10	10,4	0
Phase to phase fault bus #2	12	12,1	0

**Table 2:** Simulated fault cases.



**Figure 6:** Currents during subsequent faults.

Fig. 6 and 7 show three phase currents and voltages during different types of disturbances.



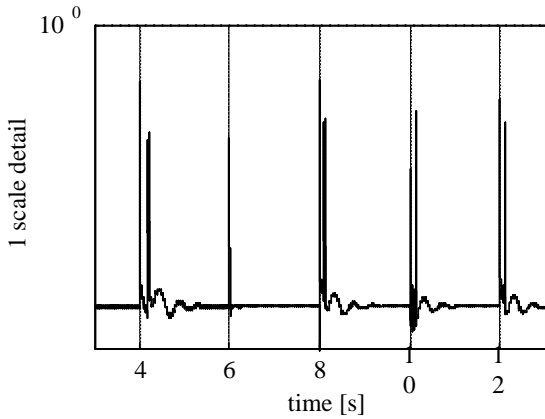
**Figure 7:** Voltages during subsequent faults.

## 5 WAVELETS APPLICATION FOR TRANSIENTS IDENTIFICATION

The Daubechies wavelets proved to be the most accurate for detection of the beginning and the end of a transient in electrical power systems.

The Daubechies wavelet was applied to compute the approximations and details for given input signals (currents and voltages) from each phase. Fig. 8 shows the mean value of all first scale details computed for three phases. Using the mean value of first scale details of all the phases instead of each one separately, enables precise detection of the beginning and the end of transients.

The time interval of a disturbance was marked by peaks on the first scale details vs. time plot. The peak levels were significantly higher than the rest of details, leaving little space for ambiguities. That allowed a simple method for windowing the events. The accuracy was up to one or two sampling intervals (0.0001 s), which allowed appropriate selection of the disturbed signals for further parameters estimation.

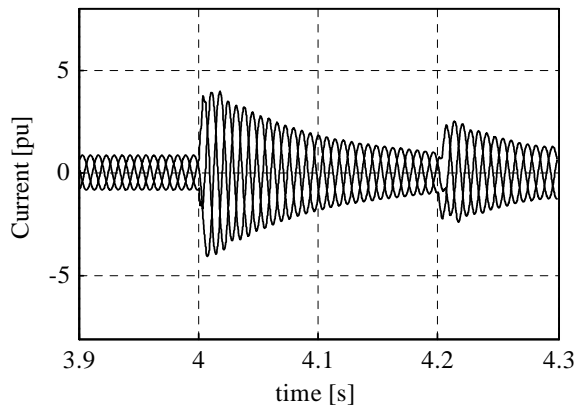


**Figure 8:** Mean value of first scale detail of currents and voltages.

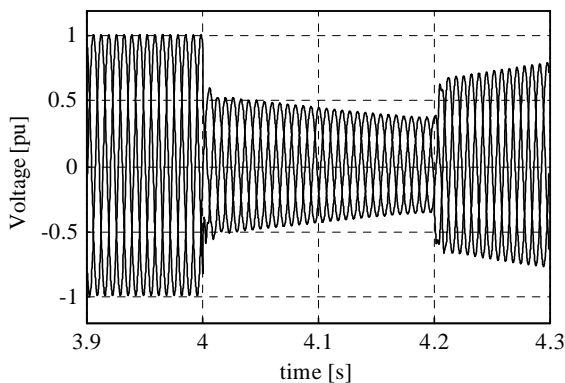
## 6 PARAMETERS OF TRANSIENTS ESTIMATED BY PRONY METHOD

### 6.1 Analysis of simulated signals

Numerous fault scenarios were investigated during the research. Fig. 9 and 10 show the currents and voltages during a three phase fault. The fault starts at 4.0001 s and ends at 4.2006 s.



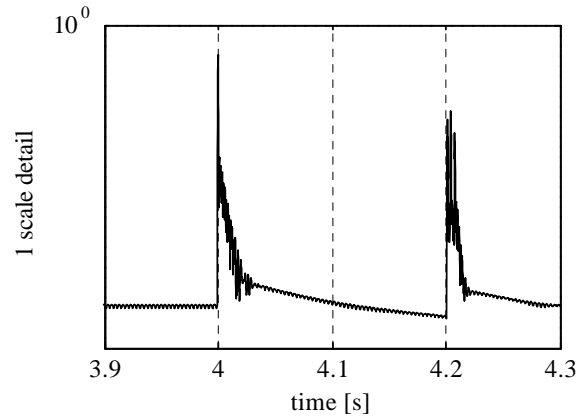
**Figure 9:** Currents during a three phase fault



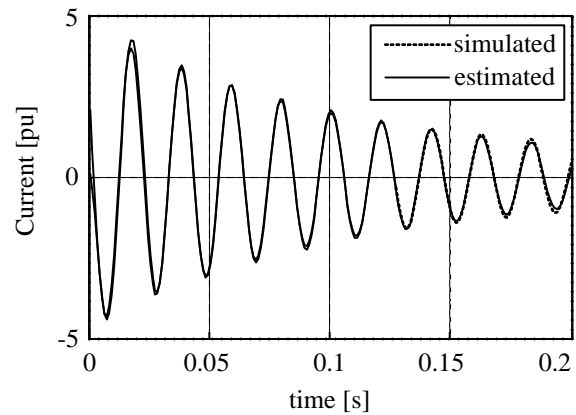
**Figure 10:** Voltages during a three phase fault.

Characteristic for the fault is exponentially decaying current. As expected, the voltage is suffering a sag during the fault. Fig. 11 shows first scale details for Daubechies function, which clearly identifies the time window of the fault. Signal parameters computed by Prony method describe the fault current in quite satisfactory

way. Little discrepancies could be seen between the simulated and estimated signal (Fig. 12). Computed signal parameters are summarized in Table. 3.



**Figure 11:** First scale details for Daubechies function

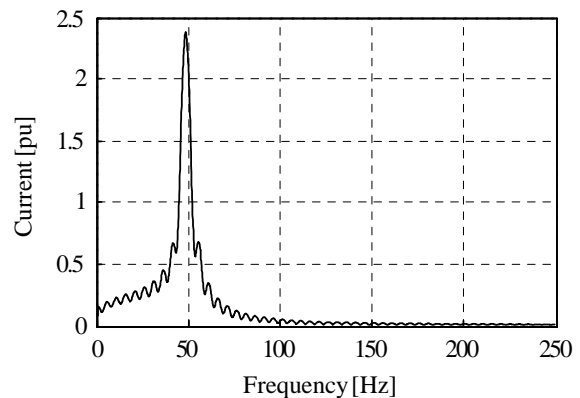


**Figure 12:** Simulated and estimated current signal

$A$ [p.u.]	$\alpha$ [1/s]	$f$ [Hz]	$\psi$ [rd]
4.42	7.88	48.2	1.09

**Table 3:** Fault current parameters obtained by Prony method.

The Fourier analysis of the fault current (Fig. 13) shows one dominant spectral component. Regarding the decaying character of the current during the fault, no exact estimation of the amplitude could be obtained by Fourier analysis. In a similar manner, analysis of other faults can be carried out.

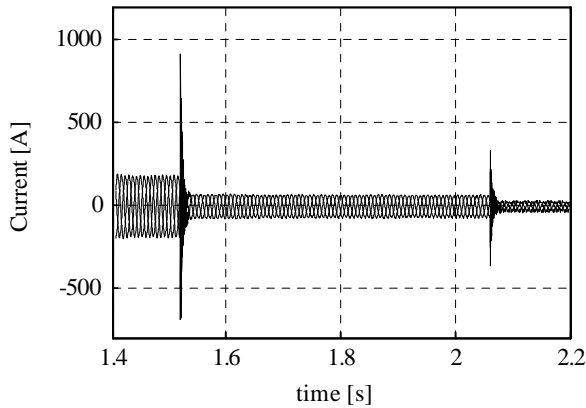


**Figure 13:** Fourier transform of the fault current.

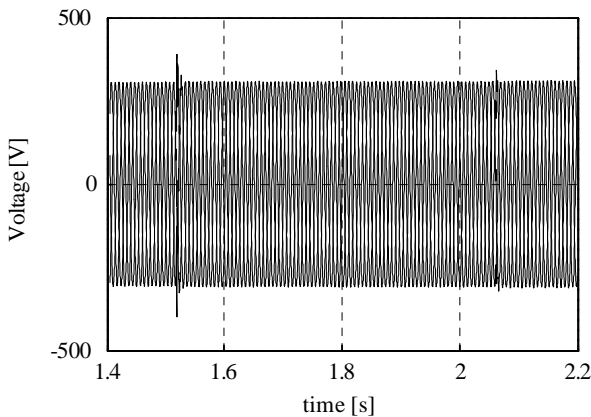
### 6.2 Analysis of measured signals

The Prony model was also applied to estimate the parameters of measured signals. The measurements were done on a system connected to a 500 kW induction generator with two step compensation capacitors. Measuring recorder was connected to the high voltage side of a 11kV/400V, Dyn11, transformer. The signal was sampled at 6250 Hz.

Currents and voltages during capacitor bank switching are shown in Figs. 14 and 15 respectively. The first capacitor was switched on at 1.52 s and the second at 2.08 s.



**Figure 14:** Currents during subsequent capacitor bank switching



**Figure 15:** Voltages during subsequent capacitor bank switching

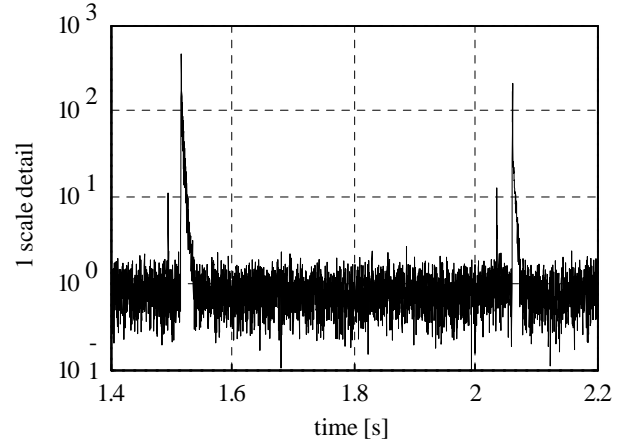
The most accurate detection of the beginning of a transient event has been achieved for 5 coefficients of a Daubechies wavelet filter. The first scale detail signal enabled detection of transients with accuracy 1 or 2 sampling intervals. Mean value of first scale details of these signals are shown in Fig. 16. The transients resulting from both switching operations were analyzed. The windowed current waveform showing the first switching transient in detail is shown in Fig. 17. Fig. 18 depicts the Fourier spectrum of this signal.

Table 4 includes the signal parameters obtained by Prony method. Frequency values correspond with spectral components visible in the Fourier spectrum (Fig. 17). Prony method, in contrary to Fourier analysis, delivered accurate initial value of amplitude of decaying

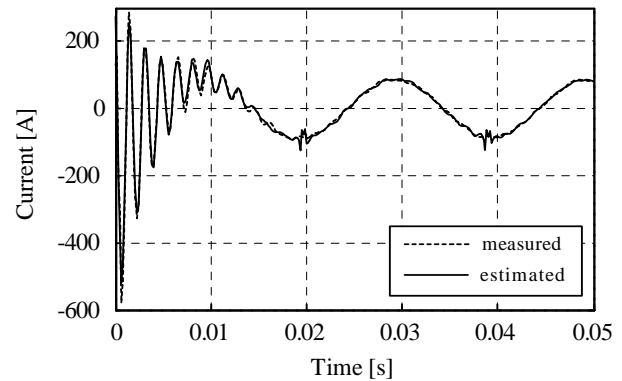
component. Higher frequency sinus wave is multiplied by an exponential decaying component (Fig. 17). That's why amplitudes about 597 Hz are so different in both methods. The windowed current waveform showing the second switching transient is shown in Fig. 19.

k	I [A]	$\alpha$ [1/s]	f [Hz]	$\psi$ [rd]
1	82.1	0.30	50.0	2.91
2	419	210	597	0.73

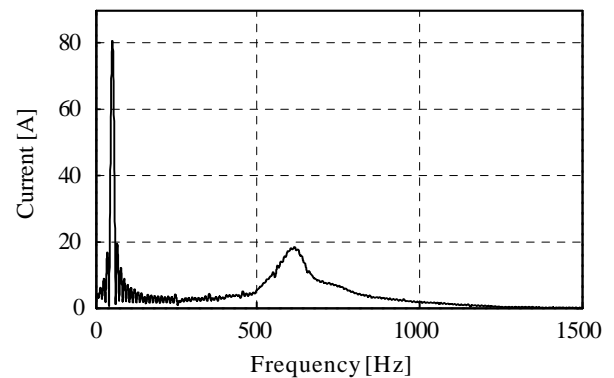
**Table 4:** Current parameters obtained by Prony method.



**Figure 16:** Scale and wavelet for Daubechies function.



**Figure 17:** Measured and estimated signal.



**Figure 18:** Fourier spectrum of the fault current

As previously, estimated signal parameters are presented in the Table 5. The Fourier spectrum (Fig. 20) indicates additional small components in the signal.

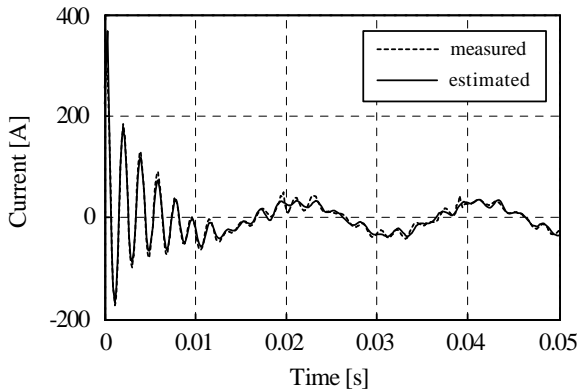


Figure 19: Measured and estimated signal.

k	I [A]	$\alpha$ [1/s]	f [Hz]	$\psi$ [rd]
1	33.1	2.09	50.6	0.61
2	259	190	534	0.85

Table 5: Current parameters obtained by Prony method.

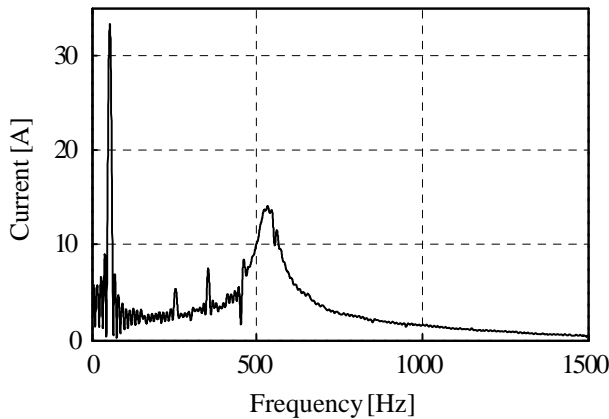


Figure 20: Fourier spectrum of the fault current

## 7 CONCLUSION

The assessment of transients in power systems with wind generation is crucial for power quality issues and for adequate proposals of disturbance mitigation systems. Research results show, that the combination of wavelet and Prony method is useful in transients' analysis.

Prony method is a parametric method, and as such predefines an exponential signal model. Fourier transform, as a nonparametric method, did not require a signal model or even the number of components, but could not compute parameters besides frequency, for non stationary signals.

Daubechies wavelet was successfully applied for detection of the beginning and the end of transient phenomena. Only properly windowed disturbing events ensured accurate computation of current and voltages signal parameters.

The Prony model proved to be adequate for transient estimation in systems with wind generators and compensating capacitors. The method enables accurate estima-

tion of amplitude, time constant, phase and frequency of transients' components of simulated and measured signals. Especially measured and noisy values proofed the applicability of Prony method in real applications. Current and voltage waveforms and its parameters vary and depend strongly on system elements and their operating mode. They should be computed for particular installations, separately.

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